Is the Clauser–Horne model of Bell's theorem completely stochastic?

2005 J. Phys. A: Math. Gen. 38 3661

(http://iopscience.iop.org/0305-4470/38/16/013)

INSTITUTE OF PHYSICS PUBLISHING JOURNAL OF PHYSICS A: MATHEMATICAL AND GENERAL

J. Phys. A: Math. Gen. **38** (2005) 3661–3664 doi:10.1088/0305-4470/38/16/013

Is the Clauser–Horne model of Bell's theorem completely stochastic?

H Razmi

Habibollah Razmi, 37185-359, Qom, I R Iran
and
Department of Physics, The University of Qom, Qom, I R Iran

E-mail: razmi@qom.ac.ir and razmiha@hotmail.com

Received 23 December 2004, in final form 7 March 2005
Published 6 April 2005
Online at stacks.iop.org/JPhysA/38/3661

Abstract
The stochastic Clauser–Horne (CH) model of Bell's theorem [1] is considered and by applying the locality condition it is shown that this (local) model, as far as applied to the singlet-state and without using quantum mechanical formalism, is not completely stochastic (i.e. there are possible configurations for which the model is deterministic).

PACS number: 03.65.Ud

Introduction

Quantum theory widely violates Bell inequality; it should be in conflict with at least one of the assumptions used in the derivation of this inequality (Bell's theorem). The main assumptions, among other possible known (e.g. counterfactual definiteness [2]) and unknown assumptions, J S Bell used in the derivation of his inequality are determinism, realism and Einstein's locality principle [3].

In 1974 Clauser and Horne made one of the most important improvements to Bell's theorem [1]. On the practical side, they derived an inequality, known as the Clauser–Horne inequality, which can be experimentally tested more feasibly. On the theoretical side, they eliminated Bell's assumption of determinism and worked with a stochastic model.

In this work, first the basic formulation of the Clauser–Horne (CH) model is reviewed. Then, by means of the locality condition, it is shown that the application of the CH model to a system in a singlet-state (without using quantum mechanical formalism) results in that this model behaves deterministically for some particular configurations. In fact, we want to check if the local form of the CH model is completely stochastic.

A brief summary of the basic formulation of the CH model

The standard Bell inequalities apply to a pair of spatially separated systems, and are written in terms of correlations between measurable quantities associated with the two systems. Consider a system which decays into two spin $\frac{1}{2}$ particles. The particles are produced in a singlet-state (total spin = 0), and go in opposite directions. Each particle goes through a Stern–Gerlach apparatus and is then detected. The Stern–Gerlach apparatus receiving particle '1' takes orientations $\hat{a}$ or $\hat{a}'$, and that receiving particle '2' takes orientations $\hat{b}$ or $\hat{b}'$. Denote by $P_1(\hat{a}, \lambda)$ and $P_2(\hat{b}, \lambda)$ the probability for the detection of particles '1' and '2', respectively, and by $P_{12}(\hat{a}, \hat{b}, \lambda)$ the probability that both particles are detected simultaneously. Here λ denotes the collection of (hidden) variables characterizing the state of each particle with a normalized probability distribution $\rho(\lambda)$

$$\int \mathrm{d}\lambda\, \rho(\lambda) = 1. \tag{1}$$

Clauser and Horne derived the following inequality:

$$-1 \leqslant P_{12}(\hat{a}, \hat{b}) - P_{12}(\hat{a}, \hat{b}') + P_{12}(\hat{a}', \hat{b}) + P_{12}(\hat{a}', \hat{b}') - P_1(\hat{a}') - P_2(\hat{b}) \leqslant 0 \tag{2}$$

where $P_1(\widehat{a})$ and $P_2(\widehat{b})$ (similarly for primed angles) are the probabilities, after averaging over probability distribution $\rho(\lambda)$, of detecting a count at the left detector (i.e. D_1) and a count at the right detector (i.e. D_2), respectively. Clearly, $P_{12}(\hat{a}, \hat{b})$ is the probability, after averaging over probability distribution $\rho(\lambda)$, of detecting a coincidence (simultaneous detection by both detectors)

$$P_1(\widehat{a}) = \int \mathrm{d}\lambda\, \rho(\lambda) P_1(\hat{a}, \lambda) \tag{3}$$

$$P_2(\widehat{b}) = \int \mathrm{d}\lambda\, \rho(\lambda) P_2(\hat{b}, \lambda) \tag{4}$$

$$P_{12}(\hat{a}, \hat{b}) = \int \mathrm{d}\lambda\, \rho(\lambda) P_{12}(\hat{a}, \hat{b}, \lambda). \tag{5}$$

The inequality (2) is the CH version of the Bell inequality. In deriving this inequality, Clauser and Horne used the following locality condition:

$$P_{12}(\hat{a}, \hat{b}, \lambda) = P_1(\hat{a}, \lambda) P_2(\hat{b}, \lambda) \tag{6}$$

to ensure that there is no action at a distance between instrument(s) 1 and instrument(s) 2.

Clearly, the CH model is a stochastic realistic model.

Is the CH model completely stochastic?

For the anti-parallel configuration of both sides of the apparatus considered in this model, we have

$$P_{12}(\hat{a}, \hat{b}) = \tfrac{1}{2} \qquad \text{for} \quad \theta_{ab} = (\theta_a - \theta_b) = \pi \tag{7}$$

where it should be mentioned that although one can simply find the above result from quantum theory, here we are not to use quantum mechanics at all and the above equality is a simple intuitively predictable result through the CH model. It is enough to consider the fact that the particles are produced in a singlet-state. Of course, pointing out the 'singlet-state' may mislead people that we have used an example from quantum mechanics and since any conclusion about the realistic model we have considered here cannot be based on an example that comes from quantum mechanics, let us explain what we mean by 'singlet-state' in this work: a completely correlated composite system often used in the experiments on Bell's theorem such as that in a proton–proton scattering experiment [4], or that in a polarization correlated state of two photons comes from annihilation of a positronium atom and so on [5]. We all know that there are many real phenomena and experiments; for them we apply separately quantum mechanical results and hidden variable realistic (here CH) model results in proving Bell's theorem. Almost in all these real phenomena/experiments one deals with a completely correlated composite system (quantum mechanically called a singlet-state). Here by 'singlet-state' we mean such a system. If we use a formula from quantum mechanics and/or a statement of quantum mechanical formalism about the system we have called 'singlet-state', then our explanation could have been unreasonable. As is clear through all parts of the work, we have not used quantum mechanical formalism anywhere.

Now, by using (7), locality condition (6) and definition (5), we arrive at the result

$$\int \mathrm{d}\lambda\, \rho(\lambda) P_1(\widehat{a}, \lambda) P_2(\widehat{b}, \lambda) = \frac{1}{2} \qquad \text{for} \quad \theta_{ab} = \pi. \tag{8}$$

On the other hand

$$\int \mathrm{d}\lambda\, \rho(\lambda) P_1(\hat{a}, \lambda) = P_1(\widehat{a}) = \frac{1}{2} = P_2(\widehat{b}) = \int \mathrm{d}\lambda\, \rho(\lambda) P_2(\hat{b}, \lambda) \qquad \text{for} \quad \theta_{ab} = \pi \tag{9}$$

where, again, we do not need to use (and have not used) quantum theoretical predictions (in addition to the above explanation after equation (7), we can also refer to the experimental results in justifying (7) and (9)).

Comparison of (9) and (8) leads to

$$\int P_1(\widehat{a}, \lambda)[1 - P_2(\widehat{b}, \lambda)]\, \mathrm{d}\lambda\, \rho(\lambda) = 0$$
$$= \int P_2(\widehat{b}, \lambda)[1 - P_1(\widehat{a}, \lambda)]\, \mathrm{d}\lambda\, \rho(\lambda) \qquad \text{for} \quad \theta_{ab} = \pi. \tag{10}$$

All integrands of the above integrals are positive definite ($\rho(\lambda) \geqslant 0$, since it is a probability weighting function and $0 \leqslant P_1(\widehat{a}, \lambda)$, $P_2(\widehat{b}, \lambda) \leqslant 1$); thus, the equalities in (10) are impossible unless

$$P_1(\widehat{a}, \lambda)[1 - P_2(\widehat{b}, \lambda)]\rho(\lambda) = P_2(\widehat{b}, \lambda)[1 - P_1(\widehat{a}, \lambda)]\rho(\lambda) = 0 \qquad \text{for} \quad \theta_{ab} = \pi. \tag{11}$$

Clearly, $\rho(\lambda) = 0$ corresponds to trivial cases that can be excluded just at the first phase of calculations. In other words, if $\rho(\lambda)$ vanishes for all values of λ then we can conclude from this fact that the model is trivial and not a realistic (dealing with hidden variables) model at all; but if only one case occurs for which $\rho(\lambda) \neq 0$, it will be mathematically/logically enough to conclude that there are possible cases for which we have

$$P_1(\widehat{a}, \lambda)[1 - P_2(\widehat{b}, \lambda)] = P_2(\widehat{b}, \lambda)[1 - P_1(\widehat{a}, \lambda)] = 0 \qquad \text{for} \quad \theta_{ab} = \pi. \tag{12}$$

Therefore

$$P_1(\widehat{a}, \lambda) = P_2(\widehat{b}, \lambda) = 0 \text{ or } 1 \qquad \text{for} \quad \theta_{ab} = \pi. \tag{13}$$

Although the equality $P_1(\widehat{a}, \lambda) = P_2(\widehat{b}, \lambda)$ is not a surprising result for $\theta_{ab} = (\theta_a - \theta_b) = \pi$, this result that the probability functions $P_1(\widehat{a}, \lambda)$ and $P_2(\widehat{b}, \lambda)$ must take only 0 or 1 values is a remarkable point that reveals the deterministic behaviour of the model for the particular configuration $\theta_{ab} = \pi$.

Similar steps lead to the same (deterministic) result for the configuration $\theta_{ab} = (\theta_a - \theta_b) = 0$.

Conclusion

We have found some deterministic configurations in the local CH model of Bell's theorem; therefore this model is not completely stochastic. Since locality condition (6) has been directly used through this work, we conclude with this statement: the locality assumption in the stochastic realistic CH model makes it not completely stochastic. It opens this question: is it possible to have a local completely stochastic model? It seems the locality assumption and determinism have some common roots. From one side, determinism and causality are not independent of each other but have a fair degree of overlap. From the other side, causality and locality are not independent of each other but have an even greater degree of overlap. So our result that a local model shows some hint of deterministic behaviour is perhaps not so unexpected after all[1].

In fact, Bell's theorem is a 'hard' and 'rich' theorem whose assumptions and results are of philosophical, theoretical, experimental and even mathematical significance. We must be more careful in considering and interpreting the assumptions and results of this theorem.

Finally, we should mention that although the CH model is a well-known model of about three decades ago, to the best of our knowledge, there is no other publication of similar result(s) to the present work.

References

[1] Clauser J F and Horne M A 1974 *Phys. Rev.* D **10** 526
[2] Redhead M 1987 *Incompleteness, Nonlocality, and Realism* (Oxford: Oxford University Press) chapter 3
[3] Bell J S 1964 *Physics* **1** 195
[4] Lamehi-Rachti M and Mittig W 1976 *Phys. Rev.* D **14** 2543
[5] Kasday L R, Ullman J D and Wu C S 1970 *Bull. Am. Phys. Soc.* **15** 586
Wilson A R, Lowe J and Butt D K 1976 *J. Phys. G: Nucl. Part. Phys.* **2** 613
Bruno M, d'Agostino M and Maroni C 1977 *Nuovo Cimento B* **40** 142
There are also some more modern/exact experiments on correlations of linear polarizations of pairs of photons: Aspect A, Dalibard J and Roger G 1981 *Phys. Rev. Lett.* **47** 460
Aspect A, Dalibard J and Roger G 1982 *Phys. Rev. Lett.* **49** 1804
One of the newest experiments with detectors of high efficiencies is: Rowe M A, Kielpinski D, Meyer V, Sackett C A, Itano W M, Monroe C and Wineland D J 2001 *Nature* **409** 791

[1] There are many statements and questions on this subject of whether the science of physics, classical and/or quantum is and/or should be fundamentally deterministic/causal and/or stochastic. Is the physical locality principle just the same as the philosophical causality principle? Does chance govern the physical laws?! The interested reader can simply find many works on these subjects and questions via searching historical and conceptual monographs in the literature of classical/quantum physics and related sections of the philosophy of science.